

Re-evaluation of the capital charge in insurance after a large shock: empirical and theoretical views

Fabrice Borel-Mathurin⁵², Stéphane Loisel⁵³, and Johan Segers⁵⁴

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Abstract

Motivated by the recent introduction of regulatory stress tests in the Solvency II framework, we study the impact of the re-estimation of the tail risk and of loss absorbing capacities on post-stress solvency ratios. Our contribution is threefold. First, we build the first stylised model for re-estimated solvency ratio in insurance. Second, this leads us to solve a new theoretical problem in statistics: what is the asymptotic impact of a record on the re-estimation of tail quantiles and tail probabilities for classical extreme value estimators? Third, we quantify the impact of the re-estimation of tail quantiles and of loss absorbing capacities on real-world solvency ratios thanks to regulator data from Banque de France – ACPR. Our analysis sheds a first light on the role of the loss absorbing capacity and its paramount importance in the Solvency II capital charge computations. We conclude with a number of policy recommendations for insurance regulators.

Keywords: Insurance, Extreme Value Theory, Financial Regulation, Solvency II, Solvency Capital Requirement, Loss Absorbing Capacities, Stress Tests, Enterprise Risk Management.

JEL Codes: G01, G22, G28, G32

⁵² The views expressed in this paper are those of the authors and do not necessarily reflect those of the Autorité de Contrôle Prudentiel et de Résolution (ACPR), neither those of the Banque de France. †Email: fabrice.borel-mathurin@acpr.banque-france.fr

⁵³ Université de Lyon, Université Claude Bernard Lyon 1, Laboratoire de Sciences Actuarielle et Financière, Institut de Science Financière et d'Assurances, 50 Avenue Tony Garnier, F-69007 Lyon, France.

⁵⁴ Université catholique de Louvain, ISBA, Voie du Roman Pays 20, B-1348 Louvain-la-Neuve, Belgium.

Introduction

Modern financial regulation frameworks are designed to take into account the actual risks faced by financial institutions. This precision in evaluating the risks comes at a cost since improving accuracy tends to be pro-cyclical.⁵⁵ As a response to the potential increase of systemic risk, stress tests have increasingly become a common tool for insurance and banking supervision. In a nutshell, supervisors check the consequences of adverse shocks on the solvency, liquidity and stability assessment of undertakings. Since Basel I, financial regulation is based on the assessment of capital requirement and its coverage by undertakings. In this respect, undertakings would typically undergo assets and own-fund downfall after the simulation of the shock. Some companies pass the test and still hold enough capital after the stress test while some others do not.

This type of financial stability tests is suited for supervision. On the one hand, it helps monitor financial stability based on a horizontal and cross-sectional analysis of individual responses. On the other hand, it can include a forward-looking perspective. Moreover, some supervisors almost only rely on the outcome of such exercises.⁵⁶ Even if the use of such tests is more recent in the insurance sector than the banking sector, they come more and more on top of the agenda, see for example NAIC and EIOPA's recommendations arising after such exercises (e.g. European Insurance and Occupational Pensions Authority, 2014). Different aspects of stress test exercises need to be clarified: why stress testing? How should such exercises be organized to optimize supervision efficiency? How should the scenarios be selected and at which (quantile) level? How should the framework of the exercises be designed, e.g. which simplifying assumptions should be made?

In this study we only focus on the latter aspect with a glimpse on the European insurance stress test since those exercises are part of the more general Solvency II regulatory framework which has become fully applicable since January 2016. Since the CEIOPS quantitative impact studies performed in 2011, a consensus emerged in the European Union insurance supervisory community: the absence of Solvency Capital Requirement (SCR) reassessment after a shock was regarded as a prudent hypothesis. Indeed, it is often believed that the SCR is very likely to be smaller after

⁵⁵ See for example Gordy (2003), « A risk-factor model foundation for ratings-based bank capital rules », *Journal of Financial Intermediation*, which explains the mechanisms for the banking sector.

⁵⁶ This is for example the case of the NAIC or the FED for systemically important insurers following Dodd-Franck.

the stress test is applied than initially, for example after an adverse shock leading to a decrease in the market value of the portfolio. Keeping the SCR constant would therefore correspond to a cautious strategy.

This rationale seems natural when looking at a shock on the financial markets: if stock prices would fall by 40%, say, then a second 40% shock would only correspond to a 24% decrease with respect to the initial stock price. Besides, some countercyclical measures like the equity dampener may reinforce this phenomenon.⁵⁷

However, as far as natural or man-made catastrophes in P&C risks (“Cat P&C risks”) are concerned, if some extreme scenario occurs, then it is likely that the tail distribution of the corresponding risk has to be re-evaluated. A scenario with a return time of 150 years can, upon occurrence and after re-estimation in the light of the new data, become a scenario with a 90 year return time, as observed empirically by Mornet et al. (2016) for storm risk in France. This may of course lead to an increase in the SCR.

In addition, the loss absorbing capacities generated by deferred tax or technical provisions have limitations. After a large adverse event, these capacities may be strongly reduced, and this would lead to an increase in the SCR.

In this paper, we aim at explaining these opposite effects and quantify their combined impacts on the SCR in a simplified model and also with regulatory data. Our contribution is threefold. First, we build the first stylised model for re-estimated solvency ratio in insurance. Second, this leads us to solve a new theoretical problem in statistics: what is the asymptotic impact of a record on the re-estimation of tail quantiles and tail probabilities for classical extreme value estimators? Third, we quantify the impact of the re-estimation of tail quantiles and of loss absorbing capacities on real-world solvency ratios thanks to regulator data from ACPR featuring cases where re-computing leads to an increase in the SCR. Another striking outcome of our study is the importance of loss-absorbing capacity on solvency capital ratios.

Our paper is organized as follows. In Section 1, we explain how the Solvency Capital Requirement (SCR) is computed in Solvency II. In particular, we describe regulatory stress tests and loss absorbing capacity mechanisms. In Section 2, we present our simplified model for SCR re-estimation. Section 3 quantifies the asymptotic

⁵⁷ For more explanations on how the equity dampener is set up, see the consultation paper CP-14058 https://eiopa.europa.eu/Publications/Consultations/EIOPA-CP-14-058_ITS_Equity_dampener.pdf

underestimation when one neglects a record with a theoretical extreme value analysis point of view. In Section 4, we provide orders of magnitude of the different effects using French stress test data (relevant for the whole European Union). In the conclusion, we give some policy implications and we introduce some future research questions. This paper is an abridged version of the discussion paper (Borel-Mathurin et al, 2017) which covers the different parts with larger details.

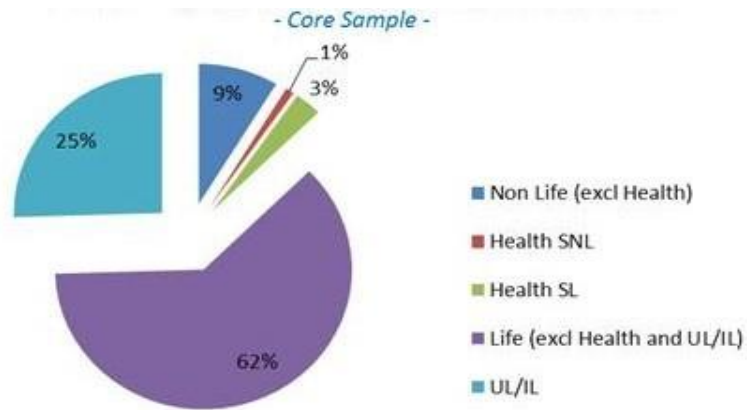
Solvency capital, stress tests and loss absorbing capacity in Solvency II: Prudential balance sheet of European insurers

In the insurance sector, estimating liabilities can be very tricky since no actual market value exists for in-force businesses. Generally, only model-based valuations are available: producing the balance sheet of an insurer is already a difficult task for life insurers, involving simulations. Technical provisions in the Solvency II framework (EU Parliament and Council, 2009) consist in an actualization of the projection of cash flows made by the undertaking. The calculation methodologies of the best estimate are defined in the Article 28 of the Delegated Regulation (Commission, 2015) and are completed in the EIOPA guidelines on Technical Provisions (European Insurance and Occupational Pensions Authority, 2015).

In 2014, EIOPA ("European Insurance and Occupational Pensions Authority") led a pan-European insurance stress test. This exercise was composed of a core exercise applied to 167 insurance groups of the EU market which included the 30 largest companies in Europe.⁵⁸ Baseline figures revealed that life technical provisions are predominant within this scope. As a consequence, market risk is actually the most important module in the aggregated SCR, see Figures 1 and 2. For this reason and to simplify the calculations, we will assume henceforth that the insurance company only depends on a single risk factor.

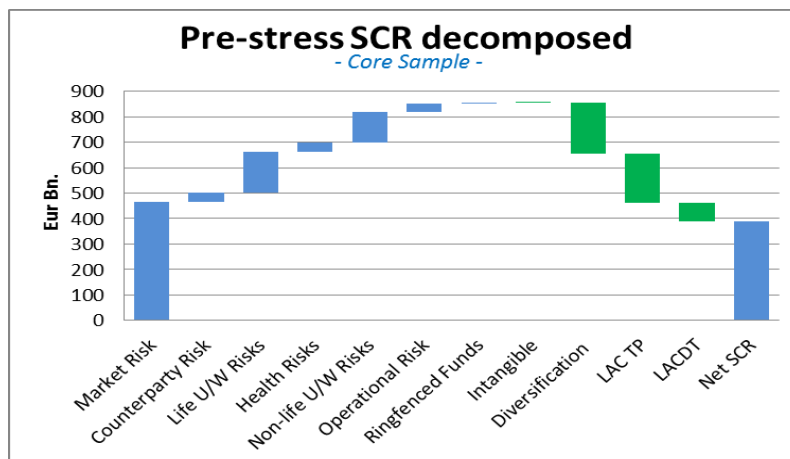
⁵⁸ This represents 55% of all gross written premiums. NCAs were allowed to add solo undertakings when unable to reach the 50% threshold with only the groups acting domestically

Figure 1: Technical provisions breakdown



Source: EIOPA Stress Test 2014

Figure 2: SCR Decomposition



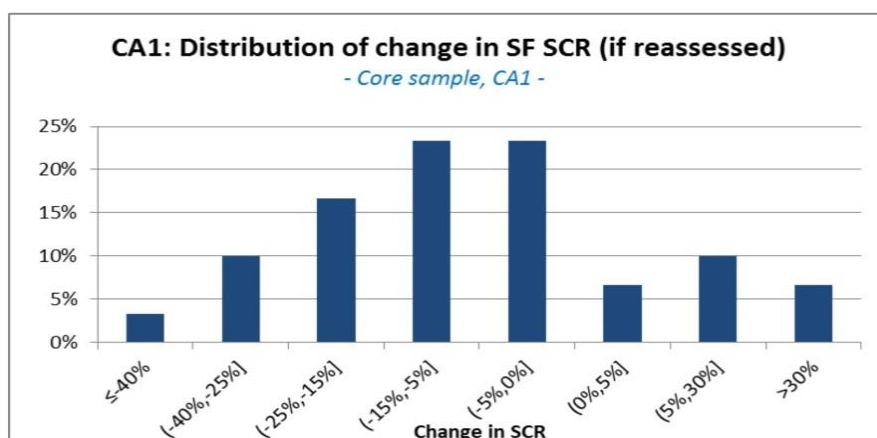
Source: EIOPA Stress Test 2014

Non-linear mitigations

Before the launch of Solvency II, CEIOPS⁵⁹ was responsible for determining which risk measure should be best suited to insurance industry.⁶⁰ Different approaches were tested for the liability valuation and already at this level the impact of the future bonuses seemed to be material.⁶¹ The insurance industry is characterized by risk mitigation and so, Solvency II, being risk based, had to take this feature into account unlike Solvency I, which was based on fixed/all-inclusive calculations. In this regard, CEIOPS progressively introduced the concept of “loss absorbing capacity” (see appendix). Considering market risk as an example: the lower the value of the assets, the lower the risk. Besides, after a large financial shock one would expect net SCR sub-modules linked to market risk to decrease when risk exposure decreases so that any SCR re-evaluation after a large shock would benefit the undertaking thanks to a proportionality effect.

However, this one-to-one correspondence is not actually observed in the 2014 Stress test data (European Insurance and Occupational Pensions Authority, 2014): although very few undertakings reassessed their SCR post-stress – less than 30%, the reassessment was optional – a significant share (more than 40%) of the undertakings underwent an increase of their global net SCR in at least one of the market scenarios.

Figure 3: Distribution of reassessed SCR



Source: EIOPA Stress Test 2014

⁵⁹ Committee of European Insurance and Occupational Pensions Supervisors, the predecessor of the European authority for insurance supervision, EIOPA

⁶⁰ The results of this analysis, called “QIS” for Quantitative Impact Studies, can be seen on the EIOPA website: <https://eiopa.europa.eu/publications/qis>

⁶¹ Approaches tested included the best estimate, the 60th, 75th and 90th percentiles, and the company view.

Indeed, taking a closer look at Figure 3, we observe that diversification effects can present some non-linearities, maybe due to the “modular” nature of their estimation. A very naive explanation to this counter-intuitive result could be that the post-stress reduction in the diversification abilities would be more significant than the reduction of risk exposure. Another simple idea would be that the addition of an extreme point changed the global shape of the underlying loss distribution. Interpretations based on both effects are developed in the following sections.

A simplified model for post-stress SCR

In this simplified model, we consider that the SCR is given by

$$SCR = [VaR_{99.5\%}(X) - E(X) - b]_+, \quad (2.1)$$

where X is a random variable corresponding to the 1-year random loss the insurer may face. Here, for simplification purposes, we consider only one risk factor, which can be financial or P&C cat. Of course, in the real world, there are many risk factors, aggregated either with the standard formula or by means of an internal model. We shall discuss the impact of diversification on our results in the sequel. The parameter b plays an important role: it corresponds to the loss absorbing capacity, and it is likely to be affected if a large event occurs.

After a shock, b is transformed into b' and X is transformed into

$$X' = a\tilde{X}, \quad (2.2)$$

where a is a factor accounting for the change in the exposure, and \tilde{X} is the revised version of X after taking the last shock into account.

If one considers mass lapse risk or pandemic risk, then the portfolio size is smaller after the first shock, so that $a < 1$. Similarly, if stock prices go down by 40%, then it is natural to consider $a = 60\% < 1$, even in absence of countercyclical measures. For P&C disasters, the situation is less clear: on the one hand, some buildings might be partly or fully destroyed, which makes the exposure temporarily decrease ($a < 1$) as there is less to be potentially destroyed by a second event. On the other hand, a first event might also cause some frailty and make the consequences of a second event potentially more severe, for example in case of floods or earthquakes where some cumulative effect or some replicas may be disastrous ($a > 1$).

If an event like a major, unprecedented earthquake, hurricane or terror attack occurs, then the probability and potential severity of such an event will automatically be re-evaluated by cat models like RMS, EQECAT or AIR or by internal models, following

Bayesian techniques. For most events, the impact on high-level Value-at-Risk is very likely to be much more important than the impact on the average. Therefore, we model this as a change from $VaR_{99.5\%}(X)$ to $VaR_{99.5\%}(\tilde{X})$, but for the sake of simplicity we do not update the average, considering that the impact on the average can be neglected: we assume that $E(X) = E(\tilde{X})$.

Of course, this assumption might be inappropriate in some cases, particularly for regime switching models like 3-state Hardy stock models or self-excited processes, in which the best estimate and the volatility tend to move in adverse directions when things go bad, and for mean-reverting models, where some mitigation is present when things go bad. For some other risks like sovereign risk or foreign exchange risk, some shocks may occur as jumps (CHF/EUR exchange rate in January 2016). The two types of risks that we consider in this paper, market shocks and large P&C claims, are thus both relevant for our study. The parameter b , accounting for the loss absorbing capacity, can be transformed into b' after a large event for several reasons. The loss absorbing capacity thanks to differed tax and thanks to technical provisions is not infinite, and it may happen that the new loss absorbing capacity after a large event is much smaller than before, that is, $b' \ll b$. Reinsurances, management action or use of countercyclical mechanisms such as the equity dampener are out of the scope of our study.

Three effects are present: the ones of a in (2.2), of b in (2.1), as well as of the tail quantile re-estimation. From a theoretical point of view, the impact of the first two ones is quite straightforward. The tail re-estimation effect, however, has not yet been studied in the literature and is a bit more technical. Therefore, in the next section, we quantify the change from $VaR_{99.5\%}(X)$ to $VaR_{99.5\%}(\tilde{X})$ after a record occurs in a P&C framework, in absence of loss absorbing capacity and for $a = 1$. As this is currently not taken into account, we formulate this as the underestimation of high quantiles when one ignores the record that has just occurred.

Pre-record estimation bias of tail estimators

Notation and framework

We take a P&C view on the random loss X underlying the SCR calibration. Let X_1, X_2, \dots be i.i.d. random variables corresponding to observations of X .⁶² For simplicity, assume that their common distribution is continuous. Denote the ascending order statistics of X_1, \dots, X_n by $X_{n:1} < \dots < X_{n:n}$.

Consider statistics of the type

$$T_n = t_n(X_1, \dots, X_n),$$

where $t_n: \mathbb{R}^n \rightarrow \mathbb{R}$ is a permutation invariant function. Think of T_n as an estimator of some tail-related quantity: a tail quantile, a return level, The statistic T_n depends on the data only through the order statistics:

$$T_n = t_n(X_{n:1}, \dots, X_{n:n}).$$

We want to understand the consequences of not re-estimating the risk distribution in a stress test associated to an extreme shock. We focus on the case where the shock is unprecedented: the very recent loss corresponds to a record, like for example the Bar-le-Duc claim in 1976 for motor third party liability or Lothar in 1999 for storm risk in France. In practice, such events might be relevant for different sub-risk-modules of Solvency II (underwriting, cat, ...) and their impact might be diluted with attritional claims during the year. To simplify, we assume here that X corresponds to the random variable whose

quantile is used to derive the Solvency Capital Requirement. We assume that at a given time instant, a record occurs: the new observation is larger than what has been observed before. When should we compute the statistic: right before or right after the record?

First, assume that the record occurs at "time" n , that is, $X_n > X_{n-1:n-1}$, or, in other words, the rank of X_n among X_1, \dots, X_n is equal to n . At a given sample size, the vector of order statistics is independent of the vector of ranks. We find that

$$[T_n \mid X_n > X_{n-1:n-1}] \sim T_n. \quad (3.1)$$

That is, computing the statistic right after a record does not lead to any distortion. Second, assume that we compute the statistic right before a record occurs.

⁶² i.i.d: independent and identically distributed

Specifically, suppose that X_{n+1} is a record: $X_{n+1} > X_{n:n}$. How does the occurrence of that event affect the distribution of T_n ?

If X_{n+1} is a record in the stretch X_1, \dots, X_{n+1} , then $X_i < X_{n+1}$ for all $i = 1, \dots, n$, and the vector of order statistics $(X_{n:1}, \dots, X_{n:n})$ is equal to the vector $(X_{n+1:1}, \dots, X_{n+1:n})$. It follows that

$$[(X_{n:1}, \dots, X_{n:n}) | X_{n+1} > X_{n:n}] \sim (X_{n+1:1}, \dots, X_{n+1:n}). \quad (3.2)$$

Equation (3.2) implies that

$$[T_n | X_{n+1} > X_{n:n}] \sim t_n(X_{n+1:1}, \dots, X_{n+1:n}). \quad (3.3)$$

Computing the statistic right before the occurrence of a record has a clear impact on its distribution: compare (3.1) and (3.3).

The size of the effect depends on the function t_n . If T_n is a tail estimator, then the impact of omitting the largest observation could be potentially quite large. We work out two relevant cases for our initial problem in the following subsections.

Tail probability estimation error

We first investigate the question of tail probability re-estimation. After an extreme event, the CEO of an insurance company could ask the cat-modelling team: "What is the return period of yesterday's event?". The cat-modellers could in fact reply: "Well, two days ago I would have answered 200 years (tail probability 1/200), but today I'd rather say 120 years!". One can imagine the CEO's reaction...

The following example quantifies the change in the tail probability estimate. Example 1 (Tail probability). Let u be a high level. Aim is to estimate the tail probability $p = 1 - F(u)$. Note that the return level is equal to $1/p$. The simplest possible estimator is the empirical one, which is clearly unbiased ($E[T_n] = p$),

$$T_n = \frac{1}{n} \sum_{i=1}^n I(X_i > u).$$

If we ignore the information that at time $n + 1$, a new record occurred and consider the case where $u = u_n \rightarrow \infty$ in such a way that $np = np_n = n\{1 - F(u_n)\} \rightarrow \tau \in (0, \infty)$, i.e., if $p \sim \tau/n$, then, calculating T_n with respect to $X_{n+1} > X_{n:n}$, the expected relative error converges to a nonzero limit:

$$\frac{1}{p} E[T_n | X_{n:n} < X_{n+1}] - 1 \rightarrow -\frac{1-e^{-\tau}}{\tau}, \quad n \rightarrow \infty \quad (3.4)$$

The asymptotic expected relative error is negative and depends on the limit of the expected number of exceedances, τ , over the level u .

Tail-quantile error estimation

The fact that a 200-year event might become a 120-year event implies that the new 200-year event is much more severe after the extreme event. Motivated by the SCR re-estimation question, we now investigate the impact of a record on tail-quantile estimators.

Example 2 (Tail-quantile estimator). Let Q be the quantile function of F . The aim is to estimate a tail quantile, $Q(1-p)$, where the tail probability, $p \in (0,1)$, is small. Assume that F is in the domain of attraction of the Fréchet distribution with shape parameter $\alpha \in (0, \infty)$. We will only use classical tools of extreme value theory. The interested reader may consult for example the book of Beirlant et al. (2006) for a presentation of the Fréchet domain of attraction. Let $\gamma = 1/\alpha$ be the extreme-value index. Let $k \in \{1, \dots, n-1\}$ be such that $p < k/n$. A common estimator is based on the approximation

$$Q(1-p) \approx Q(1-kn)\{(k/n)/p\}^\gamma.$$

On a logarithmic scale, the estimator takes the form

$$\log \hat{Q}_{n,k}(1-p) = \log X_{n:n-k} + \hat{\gamma}_{n,k} \log\{(k/n)/p\},$$

where $\hat{\gamma}_{n,k}$ is an estimator of the extreme-value index γ . Using the expression of the Hill estimator, we find that the tail quantile estimator is linear in the order statistics $Y_{n:n-k} < \dots < Y_{n:n}$, where $Y_i = \log X_i$. To evaluate the impact of ignoring a known record, let us compute the expectation of the estimator under the simplifying assumption that the random variables X_i are iid Pareto with shape parameter α , that is, $F(x) = 1 - x^{-\alpha}$ for $x \geq 1$. Equivalently, the random variables Y_i are iid Exponential with expectation equal to γ . In that case, $\log Q(1-p) = \gamma \log(1/p)$. A well-known representation of the order statistics from an exponential distribution yields

$$E[Y_{n:n-j+1}] = \gamma \left(\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{j} \right), \quad j \in \{1, \dots, n\} \quad (3.5)$$

Equation (3.5) yields the following expressions for the expectation of the estimator of the log tail quantile. Unconditionally, we have

$$E[\log \hat{Q}_{n,k}(1-p)] = \log Q(1-p) + \gamma \left(\frac{1}{n} + \dots + \frac{1}{k} - \log(n/k) \right).$$

The second term on the right-hand side converges to zero relatively quickly. In contrast, conditionally on the occurrence of a record on the next day, we have

$$E[\log \hat{Q}_{n,k}(1-p) | X_{n:n} < X_{n+1}] = (1 - a_k) \log Q(1-p) + \gamma \left(\frac{1}{n} + \dots + \frac{1}{k} - (1 - a_k) \log(n/k) \right),$$

Where $a_k = \frac{1}{k} \sum_{j=1}^k \frac{1}{j+1}$.

With this setup, the relative error occurs on the logarithmic scale, there is potentially a severe underestimation of the tail quantile: indeed,

$$(1 - a_k) \log Q(1 - p) = \log[\{Q(1 - p)\}^{1 - a_k}]$$

The relative error is thus given by $\{Q(1 - p)\}^{-a_k} \approx (1/p)^{a_k \gamma}$. The larger the tail index γ and the smaller the tail probability p , the larger the relative error. The result remains valid for the more general Pareto distribution $F(x) = 1 - (x/\sigma)^{-a}$ for $x \geq \sigma$, where $\sigma > 0$ is a scale parameter.

In the next section, we investigate the concrete impact of this phenomenon and of two other ones, risk exposure reduction and decrease in diversification elements, on real-world insurance regulatory capitals.

Illustration with real-world situations

In this section, we calibrate the three effects following two approaches: the first approach is related to actual risk levels used in financial regulations and the second one using the 2014 EIOPA stress test data of the French insurance regulator. We first provide orders of magnitude of the re-estimation effect on SCR in the insurance industry, in absence of loss absorbing capacity effect and for $a = 1$ in (2.2). Then, motivated by the design of the market risk SCR, we investigate the case where $a = 0.6$ and calibrate b in (2.1) and b' from real data. Finally, we study the case where $a > 1$ and we identify regions where one effect dominates the other one. On top of these empirical illustrations, we highlight the problem of the risk margin valuation, which strengthens our main conclusions on the SCR with a view on the whole prudential balance sheet.

Tail re-estimation effect

Parameter τ – Tail probability estimation error. In the case of a natural catastrophe, the expected number of high-threshold exceedances, $\tau = np$, belongs to a broad range of values. In the case of a stress test, τ is close to 0. It is quite common to consider $\tau = 1 \times \frac{1}{200}$ which is a typical target used in the Solvency II framework ($n = 1, p = \frac{1}{200}$).

The formula (3.4) for the expected relative error of the estimated exceedance probability due to the omission of the most recent record value as a function of τ is illustrated in Figure 4. We see that $\tau = 10$ exceedances already give a 10% misvaluation of the tail probability. If the expected number of exceedances decreases to $\tau = 1$, the relative estimation error goes up to 63%

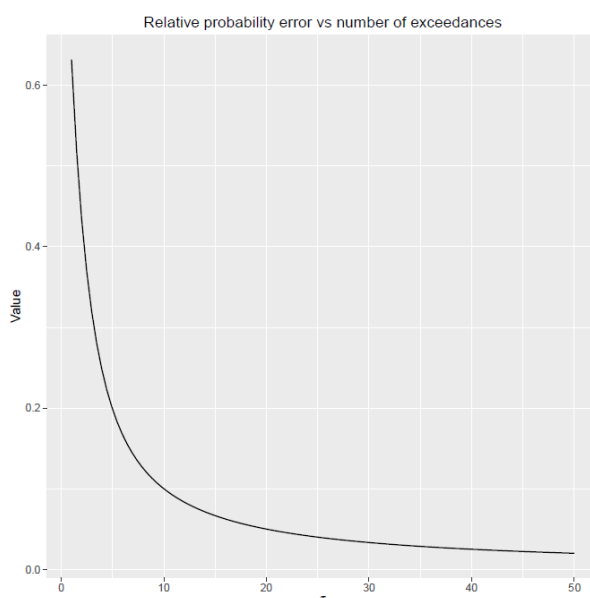
These numbers highlight the impact of the pre-record estimation bias. The effect is striking but cannot account for the error on the SCR, which is expressed on the quantile scale. We now consider the quantile error.

Parameter γ – Quantile estimation error. As a first-order approximation we can use the formula illustrated in Subsection 3.3 for the expected negative relative error of the quantile error

$$\delta_{p,k,\gamma} = (1/p)^{a_k \cdot \gamma} \quad (4.1)$$

with $a_k \approx k^{-1} \log(k)$.

Figure 4: Relative probability error vs number of exceedances estimated with (3.4) quantile estimate



Source: EIOPA Stress Test 2014

In the context of the Solvency II, p should be equal to 0.005. For k , different values are plausible; the natural framework in Solvency II should be $k = 200$, since the current norm sets records up to 200 years of magnitude. In a Stress Test context, values of k in the range of 5 to 50 are also admissible.

As illustrated by the graphs in Figure 5, the difference between the actual quantile and its value just after the addition of a shock with magnitude γ times the expectation of the standard shocks can be very significant. For example, even with 200 records, the addition of an event 10 times larger than expected would lead to a quantile more than twice the initial value! Note that we implicitly assimilated the change in the estimated 99.5% VaR to the change in the SCR. This is not true in general as the SCR might be defined in a more complex way. Besides, the Best Estimate of Liabilities

would also be impacted. Nevertheless, for reasonable values of n and k , the change in the estimated average of X is small in comparison to the change in the 99.5%-Value-at-Risk level. Therefore, for simplicity, we assume here that the Best Estimate of Liabilities can be neglected in this first study, and we leave it for further research to quantify the change in the best estimate. In Figure 6, we focus on operational risk for banks, for which banking regulation imposes to compute the 99.9%-quantile of the one-year loss. Nešlehová et al. (2006) show that for banking operational risk, one cannot exclude that $\gamma > 1$, corresponding to infinite mean models. We therefore consider the impact of quantile re-estimation after a record: for finite mean models with γ close to 1, Figure 6 shows that the new result might be as large as 2.8 times the result without re-estimation. This shows that the phenomenon presented here deserves further research regarding banking supervision.

However, this first effect actually accounts only for changes in something equivalent to the gross BSCR (the “quantile error”) before diversification (not considered here). Let us now investigate the concrete effect of Loss Absorbing Capacity on the net SCR.

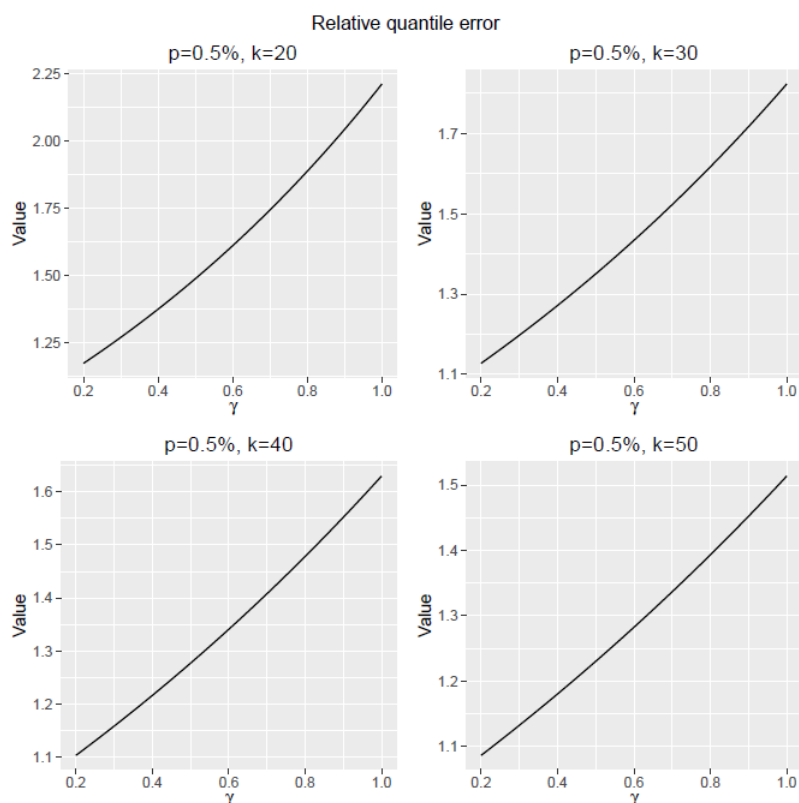
The case $a < 1$

The naive model introduced in Section 3 can be calibrated with the 2014 stress test data. An identification of the different terms on the right-hand side of (2.1) implies that $VaR_{99.5\%}(X) - E(X)$ is equal to the gross BSCR (adding Operational risk, denoted by $gBSCR$) and b is the sum of the different diversification and loss absorbing mechanisms, in particular, the Loss-Absorbing Capacity with Technical Provisions and with Deferred Tax.

In absence of quantile re-estimation, after the shock, X becomes $X' = a \cdot X$ and the SCR becomes: $SCR(X') = a \times gBSCR - b'$. With this simplified setup, it appears very clearly why the risk could not depend on the scaling factor a and only on the potential increase of volatility of the profit and loss distribution. At this point, we emphasise that the desired quantile is not directly based on the exposure so that there might only exist a tenuous link between the risk exposure and the loss distribution.

The gross SCR is multiplied by a when $X' = a \cdot X$. Note that this property is very general and remains valid when the Solvency Capital is defined via a Tail-Value-at-Risk as in the Swiss Solvency Test, or when one uses any distortion risk measure for economic capital in Enterprise Risk Management.

Figure 5: Relative quantile estimation error vs relative expected magnitude as a function of parameter γ for different values of p and k using equation 3.4

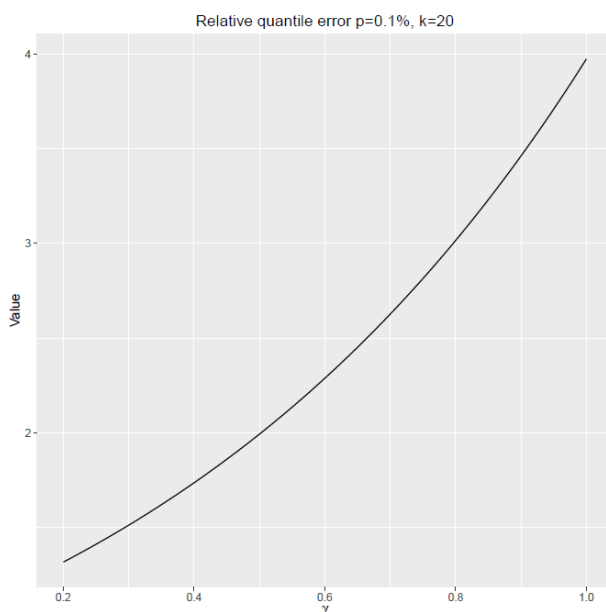


Source: Equation 3.4 in this article

This positive homogeneity property is also valid in the practical approach adopted during the genesis of Solvency II: practitioners often approximate $VaR_{99.5\%}(X)$ with $E(X) + c \cdot \sigma_X$, where σ_X is the standard deviation of X and $2.5 \leq c \leq 5$ is a multiplier close to 3 in the lognormal case and closer to 4 or 5 for loss distributions with heavier tails. To illustrate this setup, we create a company with 100Me total balance sheet representative of the ST2014 data.⁶³

⁶³ The different prudential quantities in the table are computed from the companies which reassessed their SCR post-stress and had a positive increase in at least one of the financial stress scenarios.

Figure 6: Relative quantile estimation error vs relative expected magnitude as a function of parameter γ (for $p = 0.001$ and $k = 20$) using equation (4.1)



Source: Equation 4.1 in this article

Table 1: Toy company, pre-stress situation

Liabilities	100 Me
gBSCR	7.5 Me
b	5.25Me
Net SCR	2.23Me

Source: EIOPA Insurance Stress test 2014

First remark: the diversification and loss absorbing mechanism represents more than twice the net SCR, which demonstrates its importance in the Solvency II framework.

Table 2: Toy company, post-stress situation

in M€	ST	(a ≈ a = a =	0.93)	0.9	0.8
Liabilities'	97.5	96.8	86		
BSCR'	7.17	6.7	6		
<i>b'</i>	4.45	4.02	3.27		
Net SCR'	2.71	2.71	2.71		

Source: EIOPA Insurance Stress test 2014 and authors' calculations

Another important consequence is that the variance of the profit & loss distribution plays a far greater role than the market risk exposure. Indeed, the a factor does not show up in the final estimation of the SCR. If we make another assumption and assume a perfect correlation between market exposure and the P&L, we would get:

$$SCR(X') = a \times gBSCR - b',$$

with⁶⁴ $a = 0.6$. In this simple model, the pre-stress net and gross SCR shown in Table 1 evolve⁶⁵ after the stress as presented in Table 2.

In fact, $a = 0.6$ corresponds to the pure shock for stocks and their spillovers. But given other risk modules and diversification and loss absorbing mechanisms it might be more consistent to choose⁶⁶ $a = 0.8$ or $a = 0.9$. We also provide numbers for $a = 0.8$ and for $a = 0.9$.

For the completeness of the analysis, the value of b_0 is deduced with the following equation (for $a = 0.9$, in M€):

$$b' = bBSCR' - NetSCR' = 7.17 - 2.71 = 4.45 = 0.77 \times b.$$

As discussed in Subsection 2.2, we observe in this simple example that the different diversification and loss absorbing mechanisms had to decrease much faster than the risk exposure. As a matter of fact, a reassessment of the SCR and at least the

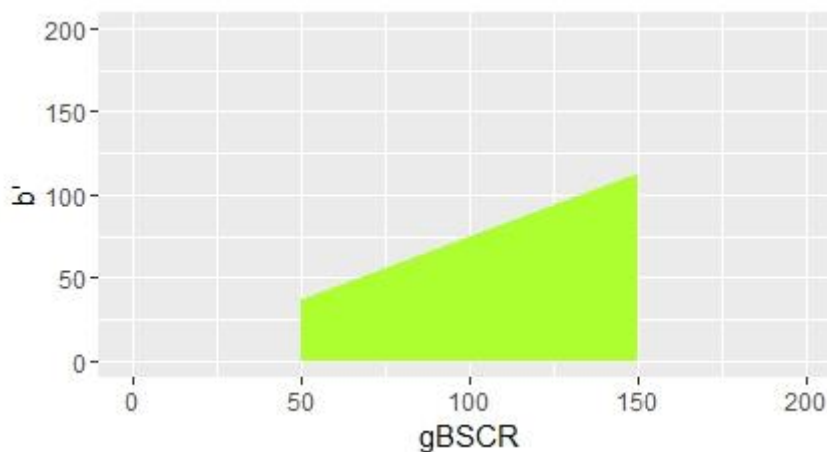
⁶⁴ This corresponds to a 40% decrease of the value of stock, comparable to the shock of the first scenario of the 2014 Stress test.

⁶⁵ The value of the LAC post-stress and BSCR' were not requested in the Stress Test exercise but could be reconstituted.

⁶⁶ As an illustration, the value was $a = 0.93$ for the French companies in the ST2014 sample used here.

different LAC component should be mandatory in any forward looking exercise (ORSA, Stress test, etc.) when it is relevant. More generally, credibility of the different diversification modules should be checked thoroughly and be part of the annual risk review of any insurance supervisor. It is interesting to note that in the case of the French groups participating to the EIOPA Stress Test 2014 which reassessed their FDB post-stress, we have $\hat{b}' = 0.26 \cdot b$,

Figure 7: : b' value with a positive increase of the net SCR



Source: EIOPA Insurance Stress test 2014

which empirically validates that this effect is quite substantial and our model is not too conservative.

The case $a > 1$

The case $a > 1$ corresponds to the situation where the risk exposure increases after the shock: for example after a first earthquake or some floodings, the next event might have more severe consequences if it occurs soon, because some buildings have become more fragile or because the soil is already saturated with water. Another such situation, in the life insurance business, may occur in the case of mass non-lapse phenomenon, where remaining policyholders are more numerous than expected, for example if they benefit from a high guaranteed minimum interest rate in a low or negative interest rate context.

To illustrate this point, we choose for b a market average and $a = 1.2$. So far, this figure has been provided as a percentage of the aggregate basic solvency capital requirement both for the participants of the 2014 EIOPA ST (European Insurance and

Occupational Pensions Authority, 2014) and their French counterparts (Borel-Mathurin and Gandolphe, 2015).

The absorption capacity is $b = 38\% \times \overline{gBSCR}$. (resp. $b = 61\% \times \overline{gBSCR}$) for the whole setup of European groups participants (resp. the French groups), where we averaged over the corresponding samples. For values of gross BSCR ranging from 50% to 150% of the market average gross SCR, we plot in Figure 7 the sub-regions of the half-plane (b' , gross BSCR) where the re-evaluated SCR is larger than the initial one.

Conclusions and policy implications

The Solvency II framework is characterized by the estimation of loss quantiles based on historical data. This framework allows for diversification and loss absorbing mechanisms and absorption capacities, that is, the ability to transfer future risk to the policyholders. In this paper, we studied the implications of the records of large losses on the one hand and, on the other hand, the magnitude of diversification elements of the prudential balance sheet such as loss absorbing capacities using deferred taxes or the technical provisions.

We computed the bias of estimators of tail probabilities and high quantiles of the loss distribution if the estimation is done immediately prior to the occurrence of a new record loss. We also proposed a stylized model to reassess the solvency capital requirement after a large record. The calibration using the data of the French participants to the 2014 EIOPA Stress test confirms our theoretical arguments and showed the very prominent role of the loss absorbing capacities in the Solvency II framework. Based on our data and as far as our estimations are concerned, the decrease in the reassessment of the solvency capital requirement is in the range of 23% to 74%. One of the regular criticism addressed to the Solvency II framework is the one-year horizon used for the quantile calculations, as it could produce a lack of stability in the determination of the solvency capital requirement. In this regard, our work stresses the volatility-inducing potential of the absorption capacities. This feature emphasizes the importance of the future management actions and other means of diversification and risk mitigation while calculating the Best Estimate of the liabilities. Implications of our paper could have four facets: research, Enterprise Risk Management, supervision and regulation. As far as research is concerned, one might want to look ahead to a more advanced framework with a multi-dimensional setup. Insurance companies potentially undergo shocks from different risk factors simultaneously, the aggregation of which would introduce other effects to model. Another direction could be the use of these ideas in the banking sector, e.g. the

calculation of the capital charge with VaRs such as Market risk in the Basel III framework.

Insurers, reinsurers and captives should take into account the impact of large events on their future ability to continue business. This study shows that re-evaluating the SCR after a shock should be part of a sound Enterprise Risk Management approach of risk measurement, risk controls and risk appetite determination.

The supervision duties should be modified in comparison to what was done in the Solvency I framework. Even in the standard formula, many levers exist and can be used while producing the prudential balance sheet. In this context, supervisory work should integrate the credibility checking of the projection hypotheses. Regarding prospective exercises, be it by the firm (e.g. ORSA) or the regulator (e.g. Stress Tests), we strongly recommend to always check the evolution of the solvency capital requirements after the occurrence of a shock, since letting these requirements remain constant cannot always be seen as a conservative assumption. Indeed, we showed in this paper that the risk exposure reduction does not necessarily decrease the value of the solvency capital requirement. In this regard we would strongly recommend that future exercises do not only specify the asset side but also the liability side and give guidance on the level of risk transfer to be operated with the technical provision.

Regarding banking supervision, our theoretical analysis and Figure 6 show that the re-estimation of the quantile of the operational loss is a very important question and deserves further research.

Finally, the regulatory bodies might have a closer look at the question of the accurate level of the chosen quantile and how to account for post-stress re-evaluation in the Solvency II framework. Such a study may motivate better ways to assess the prudential balance sheet figures estimations (SCR, MCR, etc.). A plausible response could consist of regulatory prescriptions such as floors or caps on the levels of the different diversification and loss absorbing mechanisms (LAC DT/TP, reinsurance or derivatives, and others). Moreover, the dynamic nature of capital requirements argues for simple multi-period stress tests instead of instantaneous ones.

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Appendix: Loss absorbing capacities

Before the launch of Solvency II, CEIOPS⁶⁷ was responsible for determining which risk measure should be best suited to insurance industry.⁶⁸ Different approaches were tested for the liability valuation and already at the quantile levels tested, the impact of the future bonuses were material.⁶⁹ The insurance industry is characterized by risk mitigation and so, Solvency II, being risk based, had to take this feature into account unlike Solvency I, which was based on fixed/all-inclusive calculations. In this regard, CEIOPS progressively introduced the concept of “loss absorbing capacity”: at first in the QIS 2 specifications one could find the “risk absorbing proportion of $TP_{Benefits}$ ” or the “risk absorption” property of the future profit sharing only related to the discretionary nature of profit-sharing in almost all jurisdiction:

$$RPS = k \cdot TP_{Benefits}$$

assuming a linear relation between the Reduction for Profit-Sharing (RPS) and the technical provisions (TP) which relates to the future discretionary profits, and k was the risk absorbing proportion of those technical provisions. QIS 3 was only mentioning the “loss absorbing capacity” for the purpose of the valuation of contingent capital but confirmed the key role played by future bonuses granting those mechanisms some “risk absorption” abilities or properties. The QIS 2 linear relation was still mentioned but a more complex mechanism, called a “three-step approach” was introduced: for each risk sub-module two calculations should be performed: a net SCR module, denoted by $nSCR_{mod}$, and a gross one, denoted by $gSCR_{mod}$. The difference, KC_{mod} , between those two quantities is the “risk absorption ability” at the risk module level:

$$KC_{mod} = gSCR_{mod} - nSCR_{mod}$$

With this approach, the loss absorbing capacities were not assumed to be directly comparable to a specific balance-sheet element such as the with-profits technical provisions. As a consequence, this modular calculation made it unpredictable to any movement in the balance-sheet, were it on the liability or asset side. The QIS 4 specifications only refined this approach by defining more precisely what “loss

⁶⁷ Committee of European Insurance and Occupational Pensions Supervisors, the predecessor of the European Insurance and Occupational Pensions Authority, “EIOPA”

⁶⁸ The results of this analysis, called “QIS” for Quantitative Impact Studies, can be seen on the EIOPA website: <https://eiopa.europa.eu/publications/qis>

⁶⁹ Approaches tested included the best estimate, the 60th, 75th and 90th percentiles, and the company view.

absorbing capacities" (LAC) were, whether it be linked to an asset or a liability element, insisting on the role played by deferred tax (LAC_{DT}) and absorbing capacities by the technical provisions (LAC_{TP}). Finally, the Solvency II directive gave legal perspective to the concept of loss absorbing capacity in its Articles 103 and 108; Article 111 let the implementing measures give more details on how to compute those loss absorbing capacities.

For any simulated sample path used for the projection of the liabilities entering in the valuation of the best estimate, key element of the Solvency II balance sheet, an undertaking might gain or lose some risk absorbing ability. As an illustration, in the life business, depending both on the market conditions (interest rates, stock prices, etc.) and on the level of the minimum guarantees granted to the insured, the undertaking running the best-estimate simulation might gain or lose some leeway with respect to the discretionary bonuses. In the end, any of the SCR sub-modules (net) whose calculation depends on a best estimate calculation will strongly be affected by these technical provisions' absorbing mechanisms. Finally, all those submodule loss absorbing capacities coming from technical provision or future discretionary benefits are gathered at the level of the SCR to account for a global diversification effect.

How does the mitigation actually work? In QIS 1 and 2, the risk-reduction mechanisms were initially designed and conceived by all the supervisors and regulators as constant elasticities to with-profit participations. In the final version of the regulatory texts, those mechanisms are not straightforward especially for the calculation of a modular risk module (scenario-based calculations). At first, the insurance company needs to compute the SCR net of all effects, which means that the amount of the risk-mitigation techniques are taken into account in the different best-estimate evaluations (baseline and module shock) and can change on a sample path basis. Then on a second round one has to evaluate the Gross SCR. For this purpose, all the computations need to be made while assuming only the cash flows coming from the guaranteed benefits are rediscounted when the relevant scenario affects the interest rate term structure. In the gross calculation phase, the cash flows arising from the future discretionary benefits are supposed to be constant.